EE3124 — Electric Machines & Drives | Comprehensive Exam Question Bank (65 Questions + Formula Sheet)

SECTION A — CONCEPTUAL & SHORT-ANSWER QUESTIONS (Q1-type in exam)

L1 Overview

Q1. Compare a **turbogenerator** (thermal/nuclear) with a **hydroelectric generator**. Address: speed, number of poles, rotor type, physical shape, and reason for each difference.

A1.

	Turbogenerator	Hydro Generator
Speed	High — 3000 rpm (2-pole, 50 Hz)	Low — 150–300 rpm (many poles)
Poles	2 or 4	20, 40+
Rotor type	Non-salient (round/cylindrical)	Salient (projects outward)
Shape	Long shaft, small diameter	Short shaft, large diameter
Reason	High speed → round rotor avoids centrifugal stress; few poles sufficient	Slow turbine → needs many poles; large diameter accommodates them

Relation: $n_s = \dfrac{120f}{P}$. At 50 Hz, 40 poles → 150 rpm. At 50 Hz, 2 poles → 3000 rpm.

L5 Sync

Q2. (a) Define **armature reaction** in a synchronous generator. (b) State its effect on terminal voltage V_T for lagging, unity, and leading power factor loads. (c) How is it modelled in the equivalent circuit?

A2.

(a) **Armature reaction** = distortion of the air-gap magnetic field caused by the stator current when a load is connected. The stator currents create a magnetic field B_S that vectorially adds to the rotor field B_R, changing the net field.

(b) Effect on V_T:

- **Lagging PF:** B_S opposes B_R → demagnetising → V_T drops significantly
- **Unity PF:** B_S is 90° to B_R → slight distortion only → small V_T drop
- **Leading PF:** B_S aids B_R → magnetising → V_T rises (can exceed E_A)

(c) Modelled as the synchronous reactance X_S in the per-phase equivalent circuit: $E_A = V_\varphi + I_A(R_A + jX_S)$

L1/L3-L5

Q3. Explain the function of the **commutator and brushes** in a DC machine. Why do AC machines not need a commutator?

A3.

Commutator function: The armature winding of a DC machine generates alternating current as the coils rotate through the magnetic field. The commutator (segmented copper cylinder) acts as a mechanical rectifier — it switches the external connection of each coil at the moment its current reverses, so the **external current is always unidirectional (DC)**. Carbon brushes ride on the commutator segments to provide the stationary electrical connection.

Why AC machines don't need it: In AC machines (synchronous and induction), the armature winding is on the **stator** (stationary). The stator windings are directly connected to the AC supply — no switching is needed. The field is on the rotor (supplied via slip rings for synchronous machines, or induced for induction machines). Since there's no need to convert AC to DC at the output, no commutator is required.

The commutator is the main source of maintenance issues in DC machines (brush wear, sparking).

L1/L3-L5

Q4. Distinguish between **salient-pole** and **non-salient (round) pole** rotors in synchronous machines. When is each used and why?

A4.

	Salient Pole	Non-salient (Round) Pole
Construction	Poles project outward from rotor hub	Poles flush with rotor surface
Air-gap	Non-uniform (smaller under pole, larger between)	Uniform all around
Typical poles	4+ poles (up to 40+)	2–4 poles
Speed	Low speed (<1000 rpm)	High speed (1500–3600 rpm)
Application	Hydroelectric generators, diesel generators	Steam/gas turbine generators (turbogenerators)
Reason	Low speed → can fit large salient poles on big-diameter rotor	High speed → centrifugal forces demand solid, round rotor to avoid stress

L5 Sync

Q5. Explain why a synchronous motor **cannot start direct-on-line (DOL)**. Then describe **three starting methods**.

A5.

Cannot start DOL: At standstill, the rotor is stationary while the stator field rotates at n_s. The stator field "laps" the rotor twice per electrical cycle, inducing a torque that alternates direction (first +, then −). The net average starting torque is **zero**. Attempting DOL causes large inrush current with no useful acceleration.

Three starting methods:

1. **Variable Frequency Drive (VFD):** Supply frequency starts at ~0 Hz and ramps up to rated frequency, allowing the rotor to always remain in synchronism. Most modern method.
2. **Amortisseur (damper) windings:** Short-circuited bars embedded in rotor slots — acts like a squirrel-cage rotor. Motor starts as induction motor (~95% n_s), then "pulls in" to synchronous speed. Most common for direct DOL-style starting.
3. **External prime mover:** Spin rotor to synchronous speed using a separate motor, synchronize to grid as a generator, then disconnect prime mover — machine operates as motor.

L5 Sync

Q6. Explain the **V-curves** of a synchronous motor. Define underexcited and overexcited. Why would a plant install an overexcited synchronous motor even when no mechanical load is needed?

A6.

A V-curve plots $|I_A|$ vs I_F (field current) at **constant shaft load power**. It is V-shaped because:

- As I_F increases from zero: $|I_A|$ first decreases then increases → minimum at unity PF
- **Underexcited** (left of minimum): I_A lags V_φ → motor absorbs reactive power Q (acts inductively)
- **Overexcited** (right of minimum): I_A leads V_φ → motor supplies reactive power Q to the grid (acts capacitively)

Synchronous condenser application: Other loads on a factory bus (motors, transformers) are inductive — they draw lagging current and consume Q, degrading the plant power factor and incurring utility penalty charges. An overexcited synchronous motor — even with no shaft load — supplies leading current, compensating the lagging loads and improving the overall bus power factor. This is cheaper than banks of capacitors for large installations and the reactive output is controllable by adjusting I_F.

L3 DC	**Q7.** A series DC motor is accidentally **disconnected from its load** while running. Explain what happens and why this is dangerous. How does a **cumulatively compounded motor** avoid this problem?

A7.

Series motor at no-load: Field winding is in series with armature → $\Phi \propto I_a$. Speed equation:

$$\omega_m = \frac{V_a}{K_e K_f I_a}$$

As load → 0, $I_a \to 0$ (denominator approaches 0), so $\omega_m \to \infty$. The motor **over-speeds and can fly apart mechanically**, destroying bearings, windings, and potentially injuring personnel.

Rule: Never run a series DC motor without a coupled load. Never use belt drives (belt can slip off → no-load condition).

Cumulatively compounded motor: Has both a shunt field (constant, provides minimum Φ even at no-load) and a series field. At no-load: $\Phi = \Phi_{shunt} \neq 0$ → speed is limited (like a shunt motor). Under heavy load: series field adds flux, giving high torque (like a series motor). Result: safe at no-load + high starting torque.

L6 IM	**Q8.** Compare **cage rotor** and **wound rotor** induction motors in terms of construction, ability to modify performance, and applications. Why have wound-rotor motors become less common?

A8.

Feature	Cage Rotor	Wound Rotor
Construction	Bars shorted at both ends by rings	Full 3-phase winding, slip rings on shaft
Rotor circuit access	Not accessible	External resistance insertable via slip rings
Starting	High inrush current (~$6 \times I_{rated}$), moderate τ_{start}	Can achieve maximum τ at start by setting $R_2 + R_{ext} = X_{eq}$
Speed control	Only via VFD	External R changes torque-speed curve (wasteful)
Maintenance	Very low (no brushes)	Brushes + slip rings require regular maintenance
Cost	Lower	Higher
Applications	Pumps, fans, compressors, general purpose	Large cranes, mills, hoists (historically)

Why wound rotor less common: Modern variable frequency drives (VFDs) give cage motors the same speed/torque flexibility without the maintenance penalty of slip rings and brushes. A cage motor + VFD is the preferred modern solution.

L6 IM	**Q9.** Explain why an induction motor **can never reach synchronous speed**. What is **slip** and what is the typical operating slip range at full load?

A9.

Induction motor torque requires **relative motion** between the rotating stator field (at n_s) and the rotor. As $n_m \to n_s$:

→ Slip $s = (n_s - n_m)/n_s \to 0$
→ Rotor induced voltage $E_R = s \cdot E_{R0} \to 0$
→ Rotor current $I_2 \to 0$ (no EMF to drive it)
→ Induced torque $\tau_{ind} \to 0$ (no rotor field)
→ Friction losses decelerate rotor → n_m falls below n_s again

The rotor reaches a stable equilibrium where $\tau_{ind} = \tau_{load}$ at some finite slip. **Typical full-load slip: 1–5%** for small/medium motors. The motor runs just below synchronous speed.

Slip $s = \dfrac{n_s - n_m}{n_s}$ (dimensionless). s=0: synchronous (zero torque). s=1: standstill. Normal operation: $0 < s < 0.05$.

L5 Sync

Q10. State the **four conditions** for synchronising a generator to an infinite bus. Why must the incoming generator's frequency be *slightly higher*—and not equal to—the grid frequency?

A10.

Four conditions (all must be met simultaneously):

1. RMS terminal voltage of incoming generator = grid RMS voltage
2. Phase sequence of incoming generator = grid phase sequence
3. Phase angle of incoming = grid phase angle (checked with synchroscope — pointer rotating slowly clockwise)
4. Frequency of incoming generator slightly **higher** than grid frequency

Why slightly higher, not equal: If exactly equal, the machines might lock in with $\delta = 0$ → no power transfer. With slightly higher frequency, after closing the breaker the torque angle δ naturally increases from 0, the generator begins delivering real power to the grid (positive δ = generator mode). If the frequency were lower, δ would go negative and the machine would immediately absorb power as a motor, undesirably drawing current from the grid on first connection.

A synchroscope only checks one phase — phase sequence must be verified separately with a phase sequence meter.

L7 Special

Q11. Compare **BLDC (Brushless DC)** and **sinusoidal synchronous AC motors**. Both have PM rotors and 3-phase stators — what are their key operational differences? What are BLDC's advantages over conventional PM DC motors?

A11.

Feature	BLDC	Sinusoidal Sync AC Motor
Stator winding	Concentrated (trapezoidal layout)	Distributed (sinusoidal layout)
Back-EMF	Trapezoidal	Sinusoidal
Drive current	Quasi-square (2 phases on at a time)	3-phase sinusoidal
Converted power	P_{conv}=2EI = constant (ripple-free)	Constant (3-phase balance)
Torque ripple	Higher	Lower
Noise	More	Quieter
Motor equations	DC: $E_a = k\omega$, $\tau = kI$	AC phasor equations

BLDC advantages over conventional PM DC motor: no brushes/commutator → no sparking, no maintenance, no RF noise; higher efficiency (no brush drop ≈2V); longer life; higher speeds possible (>50,000 rpm); power winding on stator → better heat dissipation.

L3 DC

Q12. Explain the **two methods of controlling the speed** of a separately excited DC motor. State the speed region each applies to, the torque capability in each region, and why they cannot be combined at the same speed.

A12.

Speed equation: $\omega_m = \dfrac{V_a}{K_e\Phi} - \dfrac{R_a}{(K_e\Phi)^2}\cdot\tau$

Method 1 — Armature Voltage Control (below base speed ω_{base}):

- Reduce V_a → lower ω_m; field Φ = constant at rated
- T–ω slope unchanged; τ_{max} = constant (rated) → **constant torque region**
- Limit: increasing V_a above rated to go above ω_{base} would insulate the armature

Method 2 — Field Weakening (above base speed ω_{base}):

- Reduce I_f → lower Φ → higher ω_m; V_a = rated
- τ_{max} decreases ($\tau = K_e\Phi I_a$, Φ↓ while I_a limited) → **constant power region**
- Limit: too much field weakening → cannot maintain torque for heavy loads

Combined range: Armature voltage below ω_{base} + field weakening above ω_{base} → total speed range up to 40:1.

L6 IM	**Q13.** What is **VVVF (Variable Voltage Variable Frequency)** control for an induction motor? Explain the three operating regions and why constant V/f is used below base speed.	**A13.** VVVF (also called V/f control) uses an inverter to vary both voltage and frequency applied to the motor, controlling its speed by changing $n_s = 120f/P$. **Three operating regions:** 1. **Low frequency / Voltage boost region (f < ~5 Hz):** Stator resistance drop becomes significant relative to induced EMF ($E \approx V - IR$). Extra voltage boost applied to maintain adequate flux and prevent torque collapse. 2. **Constant torque region ($5\text{ Hz} \le f \le f_{base}$):** Maintain V/f = constant. This keeps the air-gap flux Φ = constant (since $E \approx 4.44k\Phi f$, constant V/f → constant Φ). Constant flux → constant τ_{max}. Motor can deliver rated torque at all speeds. 3. **Constant power / field-weakening region ($f > f_{base}$):** Voltage held at rated maximum; frequency continues to increase. Flux decreases ($\Phi \propto V/f$ → decreases). Torque capability falls ($\propto 1/f^2$); power ≈ constant. Equivalent to DC field weakening. **Why constant V/f:** Core flux $\Phi \propto E/f \approx V/f$. Keeping V/f constant maintains rated flux → rated torque capability. Reducing V with f prevents core saturation at low speeds; increasing V with f prevents core saturation at high speeds.
L5 Sync	**Q14.** What is **hunting** in a synchronous motor? What causes it and how is it suppressed?	**A14.** **Hunting:** An oscillation of the rotor around its synchronous (steady-state) position. It occurs when a sudden load change or disturbance causes the torque angle δ to overshoot and then oscillate about the new equilibrium value. The rotor "swings" back and forth around n_s. **Cause:** The synchronous motor is effectively a second-order underdamped system. When δ is displaced, the restoring torque ∝ sinδ acts, but with low damping the system oscillates rather than settling directly. If undamped, oscillations can grow and the motor can pull out of synchronism. **Suppression — Amortisseur (damper) windings:** Short-circuited bars in rotor slots. During steady-state operation they carry no current (no relative motion). When the rotor oscillates, there IS relative motion between rotor and the rotating stator field → currents induced in damper bars → these currents create a braking torque proportional to oscillation speed → damping effect → oscillations die out quickly. This is the same principle as eddy current braking.
L6 IM	**Q15.** Why does an induction motor always operate at **lagging power factor**? Can it ever supply reactive power to the grid?	**A15.** **Why lagging PF:** An induction motor requires magnetising current to establish flux in the air gap. This magnetising current I_M flows through the magnetising branch X_M and is **purely lagging** (90° behind V). The total stator current = active component (in phase with V, transfers real power) + magnetising component (lags 90°). The resulting total I_1 always lags V_φ. Unlike a synchronous motor, the induction motor has no DC field excitation — it must get ALL its magnetising flux from the stator supply, which always requires lagging reactive current. The power factor is typically 0.8–0.9 lagging at full load, and can fall to 0.1–0.3 lagging at no load. **Can it supply reactive power?** A standard induction motor on a standard supply: **No** — it always absorbs Q. However, a **self-excited induction generator** (with capacitors providing the magnetising Q) can deliver both P and Q to isolated loads. On a grid, an induction **generator** (run above n_s) consumes Q from the grid for magnetisation but supplies P to the grid.

L7 Special

Q16. Describe the **Switched Reluctance Machine (SRM)**: structure, operating principle, and why it has high torque ripple. State two advantages and two disadvantages vs induction motor.

A16.

Structure: Doubly-salient — both stator and rotor have projecting poles (salient poles). No permanent magnets. No rotor windings. Stator has coils wound around each salient pole. Rotor has no windings at all (just iron teeth).

Operating principle: When a stator coil is energised, the rotor tries to minimise its reluctance by rotating toward alignment with the stator poles. Torque:

$$\tau = \frac{1}{2} \cdot I^2 \cdot \frac{dL}{d\theta}$$

Coils must be switched off before full alignment (otherwise negative torque on return stroke). Electronic commutation switches coils in sequence to maintain unidirectional rotation.

High torque ripple cause: Torque is produced in discrete pulses (one per coil firing). Between pulses, output torque dips. The non-sinusoidal nature of $L(\theta)$ means torque cannot be perfectly constant. Ripple ratio can be 20–100% depending on number of poles.

Advantages: No PMs (low cost, no magnet demagnetisation risk); simple rugged rotor → high-speed and high-temperature capable.

Disadvantages: High torque ripple → vibration and acoustic noise; requires complex power converter and position sensing.

L5 Sync

Q17. What is **voltage regulation** of a synchronous generator? Write the formula. For a generator supplying a lagging PF load, will VR be positive or negative? What about a leading PF load?

A17.

Voltage regulation:

$$VR\% = \frac{V_{NL} - V_{FL}}{V_{FL}} \times 100\%$$

where V_{NL} = no-load terminal voltage (= $|E_A|$ per phase for Y-connection, with field current unchanged) and V_{FL} = full-load rated terminal voltage.

Lagging PF load: Armature reaction demagnetises → terminal voltage drops under load → $V_{NL} > V_{FL}$ → **VR > 0 (positive)**. A larger field current is needed to restore rated V_T under lagging load.

Leading PF load: Armature reaction magnetises → terminal voltage rises under load → $V_{NL} < V_{FL}$ → **VR < 0 (negative)**. Less field current (or no excitation) is needed; the machine can be self-exciting with leading loads.

A well-regulated generator has VR close to zero. Synchronous generators typically have VR of 10–30% for lagging loads.

L3+L6

Q18. Compare a **DC series motor** and a **three-phase induction motor** for use as a traction motor (e.g. in a train). What are the tradeoffs? Which is used in modern high-speed trains and why?

A18.

Feature	DC Series Motor	3-phase Induction Motor
Starting torque	Very high ($\tau \propto I^2$)	Moderate (can be increased with VFD)
Speed control	Simple (vary V_a)	Excellent with VFD (V/f control)
Maintenance	High (commutator, brushes)	Very low (cage rotor)
Weight/power density	Lower	Higher (especially with IGBT VFD)
Regenerative braking	Possible but complex	Easy with VFD (4-quadrant operation)
Operating environment	Sparking issues in dusty/flammable areas	Totally enclosed, safe anywhere
Speed range	Wide (flux control + voltage)	Very wide with modern VFDs

Modern high-speed trains (MTR, high-speed rail) use induction motors with VFDs because: no brush/commutator maintenance (critical in tunnels), higher reliability, smooth speed control, easy regenerative braking (energy recovery into grid), better performance density.

SECTION B — MAGNETIC CIRCUIT CALCULATIONS

L2 Magnetics

9. A magnetic core with air gap: $A_c = A_g = 3.8$ cm², $l_c = 30$ cm, $g = 1.5$ mm, $N = 80$ turns, $\mu_r = 3000$, $i = 1.8$ A. Find: (a) $\Re_c$ and $\Re_g$, (b) flux Φ, (c) flux linkage λ, (d) inductance L.

A19.

(a) $\Re_c = \dfrac{l_c}{\mu_0 \mu_r A_c} = \dfrac{0.30}{4\pi\times10^{-7}\times3000\times3.8\times10^{-4}} = \boxed{\mathbf{0.211\times10^6\ A{\cdot}t/Wb}}$

$\Re_g = \dfrac{g}{\mu_0 A_g} = \dfrac{1.5\times10^{-3}}{4\pi\times10^{-7}\times3.8\times10^{-4}} = \boxed{\mathbf{3.141\times10^6\ A{\cdot}t/Wb}}$

(b) $\Phi = \dfrac{N{\cdot}i}{\Re_c+\Re_g} = \dfrac{80\times1.8}{3.352\times10^6} = \boxed{\mathbf{4.30\times10^{-5}\ Wb}}$

(c) $\lambda = N{\cdot}\Phi = 80\times4.30\times10^{-5} = \boxed{\mathbf{3.44\times10^{-3}\ Wb{\cdot}turn}}$

(d) $L = \dfrac{\lambda}{i} = \dfrac{3.44\times10^{-3}}{1.8} = \boxed{\mathbf{1.91\ mH}}$

$\Re_g/\Re_c \approx 15{:}1$ → air gap dominates. Fringing neglected ($A_g = A_c$).

L2 Magnetics

0. A **three-leg core** has N=300 turns on the centre leg. Left and right outer legs: $\Re_L = \Re_R = 8\times10^6$ A·t/Wb each. Centre leg: $\Re_C = 2\times10^6$ A·t/Wb. Find current i for $\Phi_c = 1.0\times10^{-4}$ Wb and flux in each outer leg.

A20.

$\Re_L \| \Re_R = \dfrac{\Re_L}{2} = 4\times10^6$ A·t/Wb $\Re_{total} = 2\times10^6 + 4\times10^6 = 6\times10^6$

$\text{mmf} = \Phi_c \times \Re_{total} = 1.0\times10^{-4}\times6\times10^6 = 600$ A·t

$i = \dfrac{600}{300} = \boxed{\mathbf{2.0\ A}}$

$\Phi_L = \Phi_R = \dfrac{\Phi_c}{2} = \boxed{\mathbf{0.5\times10^{-4}\ Wb}}$ (symmetry)

L2 Magnetics

1. A **linear machine (moving bar)**: B=0.5 T into page, l=1 m, R=0.25 Ω, $V_B = 100$ V (battery).

(a) Initial current and force on bar when switch first closed (bar stationary)

(b) No-load steady-state speed

(c) With a 25 N opposing force, find steady-state speed and machine efficiency

A21.

(a) At start: e=0, bar stationary:

$I_{init} = \dfrac{V_B}{R} = \dfrac{100}{0.25} = 400$ A $F_{init} = BIl = 0.5\times400\times1 = \boxed{\mathbf{200\ N}}$

(b) No-load steady state: I→0 (frictionless, no load), so $e = V_B$:

$e = BLv \rightarrow v_{NL} = \dfrac{V_B}{BL} = \dfrac{100}{0.5\times1} = \boxed{\mathbf{200\ m/s}}$

(c) Steady state with F=25 N opposing:

$BIl = 25\ \text{N} \rightarrow I = \dfrac{25}{0.5\times1} = 50$ A

$e = V_B - IR = 100 - 50\times0.25 = 87.5$ V $v = \dfrac{87.5}{0.5\times1} = \boxed{\mathbf{175\ m/s}}$

$P_{in} = V_B \times I = 100\times50 = 5000$ W $P_{out} = F\times v = 25\times175 = 4375$ W

$\eta = \dfrac{4375}{5000} = \boxed{\mathbf{87.5\%}}$

L2 Magnetics

Q22. A coil of L=100 mH carries current i=5 A. (a) Find the energy stored. (b) If the core has $\mathfrak{R}=10^5$ A·t/Wb, verify using the alternate energy formula. (c) If the air gap is doubled ($\mathfrak{R}$ doubles), how does stored energy change for the same current?

A22.

(a) $W = \frac{1}{2}Li^2 = \frac{1}{2}\times 0.1\times 25 = \boxed{\textbf{1.25 J}}$

(b) $L=N^2/\mathfrak{R} \rightarrow N^2=L\times\mathfrak{R}=0.1\times 10^5=10^4 \rightarrow N=100$ turns. $\Phi=Ni/\mathfrak{R}=100\times 5/10^5=5\times 10^{-3}$ Wb:

$$W = \frac{\Phi^2\cdot\mathfrak{R}}{2} = \frac{(5\times 10^{-3})^2\times 10^5}{2} = \frac{25\times 10^{-6}\times 10^5}{2} = 1.25 \text{ J} ✓$$

(c) If $\mathfrak{R}$ doubles: $L=N^2/\mathfrak{R}$ halves → L'=50 mH. Same current i=5A:

$W' = \frac{1}{2}\times 0.05\times 25 = 0.625 \text{ J} = \boxed{\textbf{halved}}$

Air gap stores most of the magnetic energy — but increasing the gap reduces L and thus reduces stored energy at same current. The field energy density in the gap ($B^2/2\mu_0$) determines the force between pole faces.

L2 Magnetics

Q23. A toroid: N=200 turns, mean radius r=10 cm, cross-section A=4 cm², $\mu_r=2000$. Find the current i required to establish B=1 T in the core. If the core saturates (μ_r drops to 100 at B=1.5 T), find new i for B=1.5 T and explain the implication.

A23.

$B = \mu_0\mu_r\cdot\frac{Ni}{2\pi r} \rightarrow i = \frac{B\cdot 2\pi r}{\mu_0\mu_r N}$

$i = \frac{1.0\times 2\pi\times 0.1}{4\pi\times 10^{-7}\times 2000\times 200} = \frac{0.6283}{0.5027} = \boxed{\textbf{1.25 A}}$

At saturation (B=1.5 T, $\mu_r=100$):

$i = \frac{1.5\times 2\pi\times 0.1}{4\pi\times 10^{-7}\times 100\times 200} = \frac{0.9425}{2.513\times 10^{-3}} = \boxed{\textbf{37.5 A}}$

Implication: To increase B by only 50% (from 1.0 to 1.5 T), the required current increases 30× (from 1.25 to 37.5 A). Once saturated, the core is nearly as hard to magnetise as air. This is why operating machines above rated flux is very inefficient — huge current with little flux gain. Also, reluctance $\mathfrak{R} = l/(\mu_0\mu_r A)$ becomes much larger when saturated.

SECTION C — DC MACHINE CALCULATIONS

L3 DC

Q24. A DC series motor draws 35 A at 220 V at 1000 rpm. $R_a=0.25\ \Omega$, $R_f=0.15\ \Omega$, $P_{rot}=600$ W. Find: (a) E_a, (b) developed torque τ_d, (c) output power P_{out}.

A24.

(a) $E_a = V_T - I_a(R_a+R_f) = 220-35\times 0.4 = \boxed{\textbf{206 V}}$

(b) $\omega_m=2\pi\times 1000/60=104.7$ rad/s $\quad \tau_d=\frac{E_aI_a}{\omega_m}=\frac{206\times 35}{104.7}=\boxed{\textbf{68.9 N·m}}$

(c) $P_{out}=220\times 35-35^2\times 0.4-600=7700-490-600=\boxed{\textbf{6610 W}}$

L3 DC

Q25. A 25 kW, 125 V separately excited DC machine, $E_a=127$ V, $R_a=0.03\ \Omega$, n=3000 rpm constant. Determine mode (motor/generator), I_a, terminal power, and electromagnetic torque for:
(a) $V_T=130$ V (b) $V_T=124$ V

A25.

(a) $V_T=130 > E_a=127$ → **Motor**. External supply drives current in.

$I_a=\frac{130-127}{0.03}=\boxed{\textbf{100 A}}$ $\quad P_{term}=130\times 100=13$ kW (input) $\quad P_{conv}=127\times 100=12.7$ kW

$\tau=\frac{12700}{100\pi}=\boxed{\textbf{40.4 N·m}}$

(b) $V_T=124 < E_a=127$ → **Generator**. Machine pushes current out.

$I_a=\frac{127-124}{0.03}=\boxed{\textbf{100 A}}$ $\quad P_{term}=124\times 100=12.4$ kW (output) $\quad \tau=\boxed{\textbf{40.4 N·m}}$

Same torque magnitude — what changed is the direction of power flow, not the magnitude of electromagnetic interaction.

L3 DC	**Q26.** A **DC shunt generator**: V_T=240 V, R_a=0.08 Ω, R_f=120 Ω, n=1500 rpm, load current I_L=100 A. Find: (a) field current I_f, (b) armature current I_a, (c) generated E_a, (d) input torque τ_{in}, (e) efficiency η.	**A26.** (a) $I_f = \frac{V_T}{R_f} = \frac{240}{120} = \boxed{2\text{ A}}$ (b) $I_a = I_L + I_f = 100+2 = \boxed{102\text{ A}}$ (generator: I_a flows out) (c) Generator KVL: $E_a=V_T+I_aR_a=240+102\times0.08=\boxed{248.2\text{ V}}$ (d) $\omega_m=2\pi\times1500/60=50\pi$ rad/s $\tau_{in}=\frac{E_aI_a}{\omega_m}=\frac{248.2\times102}{50\pi}=\boxed{161.4\text{ N·m}}$ (e) $P_{out}=V_T\times I_L=240\times100=24000$ W $P_{in}=E_a\times I_a=248.2\times102=25316$ W $\eta=\frac{24000}{25316}=\boxed{94.8\%}$
L3 DC	**Q27.** A DC motor: V_T=220 V, R_a=0.4 Ω, rated I_a=50 A. To limit starting current to **twice rated**, an external resistance R_{ext} is inserted in series with the armature at start. (a) Find the required R_{ext} (b) As the motor accelerates and E_a rises to 100 V, what is the new armature current with R_{ext} still in circuit? (c) At what E_a should R_{ext} be removed?	**A27.** (a) Max allowed I_a=2×50=100 A. At start, E_a=0: $I_{max}=\frac{V_T}{R_a+R_{ext}}=100\text{ A} \rightarrow R_{ext}=\frac{220}{100}-0.4=\boxed{1.8\ \Omega}$ (b) When E_a=100 V (R_{ext} still in): $I_a=\frac{V_T-E_a}{R_a+R_{ext}}=\frac{220-100}{0.4+1.8}=\frac{120}{2.2}=\boxed{54.5\text{ A}}$ (c) Remove R_{ext} when I_a without R_{ext} would not exceed 100 A: $I_a=\frac{V_T-E_a}{R_a}\le100\text{ A} \rightarrow V_T-E_a\le40\text{ V} \rightarrow E_a\ge\boxed{180\text{ V}}$
L3 DC	**Q28.** A DC shunt motor: $K_e\Phi$=1 V·s/rad, R_a=0.25 Ω, V_a=250 V, currently E_a=245 V. (a) Find no-load speed. (b) Field flux decreases by 10% ($K_e\Phi$=0.9). Find new no-load speed and explain the cause-effect chain.	**A28.** (a) $\omega_0=\frac{E_a}{K_e\Phi}=\frac{245}{1.0}=\boxed{245\text{ rad/s}}$ (b) $\omega_{0,new}=\frac{V_a}{K_e\Phi_{new}}=\frac{250}{0.9}=\boxed{277.8\text{ rad/s (13.4\% faster)}}$ Cause-effect: $\downarrow I_f \rightarrow \downarrow\Phi \rightarrow \downarrow E_a = K_e\Phi\omega \rightarrow \uparrow I_a=(V_a-E_a)/R_a \rightarrow \uparrow\tau_{ind}=K_e\Phi I_a$ (net increase since I_a rises more than Φ falls) → rotor accelerates → $\uparrow E_a \rightarrow \downarrow I_a$ → new equilibrium at higher ω.

L3 DC

Q29. A DC series motor supplies a fan where torque $T \propto n^2$. At V_T=220 V, I_a=25 A, n=300 rpm. R_a=0.6 Ω, R_f=0.4 Ω. Fan speed to be reduced to 200 rpm by inserting external resistance R_{ext}.

(a) Find developed torque T_1 and new torque T_2

(b) Find new armature current I_{a2} and R_{ext}

(c) Find new output power

A29.

(a) $E_{a1}=220-25\times1=195$ V $\omega_1=2\pi\times300/60=10\pi=31.4$ rad/s

$$T_1=\frac{E_{a1}I_{a1}}{\omega_1}=\frac{195\times25}{10\pi}=\boxed{155.2\ \text{N·m}}$$

$$T_2=T_1\times(200/300)^2=155.2\times0.444=\boxed{68.9\ \text{N·m}}$$

(b) Series motor: $T\propto I_a^2 \rightarrow T_2/T_1=(I_{a2}/I_{a1})^2$:

$$I_{a2}=25\times\sqrt{(68.9/155.2)}=25\times\sqrt{0.444}=25\times0.667=\boxed{16.67\ \text{A}}$$

$$E_{a2}=E_{a1}\times\frac{I_{a2}\times n_2}{I_{a1}\times n_1}=195\times\frac{16.67\times200}{25\times300}=195\times0.444=86.6\ \text{V}$$

$$R_{ext}=\frac{V_T-E_{a2}}{I_{a2}}-(R_a+R_f)=\frac{220-86.6}{16.67}-1.0=8.0-1.0=\boxed{7.0\ \Omega}$$

(c) $P_{out,2}\approx E_{a2}\times I_{a2}=86.6\times16.67=\boxed{1443\ \text{W}}$

L3 DC

Q30. Compare **armature voltage control** vs **armature resistance control** for speed reduction of a DC motor. Why is resistance control considered inefficient? Calculate power loss in R_{ext}=2 Ω with I_a=40 A and compare to useful output if E_a=100 V at that operating point.

A30.

Feature	Armature Voltage Control	Armature Resistance Control
Method	Reduce V_a via converter	Insert R_{ext} in series with armature
Efficiency	High (≈95%+ for modern converters)	Low (energy wasted in R_{ext})
Speed range	Wide, precise	Limited by heat dissipation
Heat generated	Minimal	Large (proportional to I^2R_{ext})
Speed regulation	Good (T–ω slope unchanged)	Poor (T–ω slope steepens)
Initial cost	Higher (needs converter)	Cheaper (just a resistor)

Efficiency calculation:

$P_{\text{wasted in } R_{ext}}$

$= I_a^2\times R_{ext}$ = $40^2\times2$ = 3200 W

$P_{useful} = E_a\times I_a$ = 100×40 = 4000 W

Resistor wastes $\boxed{3200/7200 = 44\%}$ of total input — clearly inefficient.

SECTION D — AC MACHINE FUNDAMENTALS

L4 AC Fund.

Q31. A 2-pole, 3-phase generator: B_M=0.2 T, n=3600 rpm, stator diameter=0.5 m, l=0.3 m, N_C=15 turns/coil, Y-connected. Find: (a) three phase voltages, (b) $V_{\varphi,rms}$, (c) $V_{T,rms}$.

A31.

$\omega_m=120\pi$ rad/s $\Phi_M=2\times0.25\times0.3\times0.2=0.03$ Wb $\hat{e}=15\times120\pi\times0.03=54\pi=$**169.6 V**

(a) $v_a=169.6\sin(120\pi t)$ V, $v_b=169.6\sin(120\pi t-120°)$ V, $v_c=169.6\sin(120\pi t-240°)$ V

(b) $V_\varphi=169.6/\sqrt{2}=\boxed{119.9\ \text{V}\approx120\ \text{V}}$ (c) $V_T=\sqrt{3}\times120=\boxed{207.8\ \text{V}\approx208\ \text{V}}$

L4 AC Fund.

Q32. (a) A synchronous machine must produce 50 Hz power at exactly 375 rpm. How many poles P must it have? (b) A 6-pole machine in a 60 Hz system — what is its synchronous speed? (c) A machine has n_s=600 rpm on a 50 Hz system — how many poles? Is this physically realizable?

A32.

(a) $P=\frac{120\times f}{n_s}=\frac{120\times50}{375}$ = 16 poles $\boxed{\textbf{P = 16 (8 pole pairs)}}$ ✓ (integer, even)

(b) $n_s=\frac{120\times60}{6}=\boxed{1200\ \text{rpm}}$

(c) $P=\frac{120\times50}{600}$ = 10 poles $\boxed{\textbf{P=10 (5 pole pairs) — physically realizable}}$

Key check: P must be a positive even integer. Odd-pole machines cannot be built (poles must come in N-S pairs).

L4 AC Fund. **Q33.** A four-pole stator winding has an electrical angle θ_e=180°. (a) What is the mechanical angle θ_m? (b) If the supply frequency is 50 Hz, what is the mechanical speed n_s? (c) A fractional-pitch coil spans 160° electrical instead of 180°. Qualitatively, what is the benefit and the cost?	**A33.** (a) $\theta_m = \frac{\theta_e}{P/2} = \frac{180°}{2} = \boxed{\textbf{90° mechanical}}$ (b) $n_s = 120 \times 50/4 = \boxed{\textbf{1500 rpm}}$ (c) **Benefit of fractional pitch (160° vs 180°):** The pitch factor k_p = sin(pitch/2) = sin(80°) = 0.985. The coil's fundamental voltage is slightly reduced (by 1.5%) but harmonic voltages (3rd, 5th, etc.) are much more reduced — e.g. 3rd harmonic has pitch factor sin(3×80°)/sin(3×90°) = sin(240°)/1 ≈ 0.866/1 — significantly attenuated. Result: cleaner, more sinusoidal output voltage. **Cost:** Fundamental voltage is slightly reduced (by factor k_p < 1), and end turns are slightly longer. But the reduction in harmonic distortion usually outweighs this.
SECTION E — SYNCHRONOUS GENERATOR	
L5 Sync **Q34.** A 200 kVA, 480 V, 50 Hz, Y-connected synchronous generator. At rated I_F=5 A: $V_{T,OC}$=540 V, $I_{L,SC}$=300 A. DC test: V_{DC}=10 V, I_{DC}=25 A. Find R_A, E_A, and X_S.	**A34.** $R_A = \frac{V_{DC}}{2I_{DC}} = \frac{10}{50} = \boxed{\textbf{0.20 Ω}}$ $E_A = \frac{540}{\sqrt{3}} = \boxed{\textbf{311.8 V}}$ $Z_S = \frac{311.8}{300} = 1.039\ \Omega$ $X_S = \sqrt{1.039^2 - 0.20^2} = \boxed{\textbf{1.020 Ω}}$
L5 Sync **Q35.** A 6 kV, 3-phase, Y-connected generator, X_S=0.6 Ω, R_A≈0, supplying 30 MVA at 0.9 PF lagging. Find: (a) V_φ, I_A, (b) E_A and δ, (c) voltage regulation VR%.	**A35.** (a) $V_\varphi = 6000/\sqrt{3} = 3464$ V $I_A = 30\times10^6/(\sqrt{3}\times6000) = 2887$ A∠−25.84° (b) E_A=3464+j0.6×2887∠−25.84°=3464+1732∠64.16°=4217+j1559 $\lvert E_A\rvert = \boxed{\textbf{4498 V}}$ $\delta = \tan^{-1}(1559/4217) = \boxed{\textbf{20.3°}}$ (c) $VR\% = \frac{\lvert E_A\rvert - V_\varphi}{V_\varphi} \times 100 = \frac{4498-3464}{3464} \times 100 = \boxed{\textbf{29.9\%}}$ *High VR for lagging load — field current must increase significantly to maintain rated V_T under load.*
L5 Sync **Q36.** Same generator as Q35, but now supplies 30 MVA at 0.9 PF **leading**. Find E_A and VR%. Compare with Q35 result and explain physically.	**A36.** I_A=2887∠+25.84°=2598+j1259 A (leading) E_A=3464+j0.6×(2598+j1259)=3464+j1559+j²×755=3464−755+j1559=2709+j1559 $\lvert E_A\rvert = \sqrt{2709^2+1559^2} = \boxed{\textbf{3125 V}}$ $VR\% = \frac{3125-3464}{3464} \times 100 = \boxed{\textbf{−9.8\% (negative!)}}$ **Physical explanation:** Leading current means the stator magnetic field **aids** the rotor field (magnetising armature reaction). The terminal voltage RISES when this load is connected. If the load disconnects, V_T would **fall** to V_{NL}=|E_A| =3125 V < 3464 V. Capacitive loads can be dangerous — generators may overvoltage on load rejection if they were pre-compensating for the leading power factor.

L5 Sync **Q37.** A 100 MVA, 14.4 kV, 0.8 PF lagging, 2-pole, 50 Hz, Y-connected generator: X_S=2.281 Ω, R_A=0.0228 Ω. Find: (a) $\lvert E_A \rvert$ and δ, (b) shaft input torque (ignore losses), (c) what is the maximum power this generator can deliver if E_A stays at this value?	**A37.** $V_\varphi=14400/\sqrt{3}=8314$ V $I_A=100\times10^6/(\sqrt{3}\times14400)=4009\ A\angle-36.87°$ (a) $E_A=8314+(3207-j2405)(0.0228+j2.281)=13873+j7261$ $\lvert E_A\rvert=\boxed{15{,}658\ V}$ $\delta=\boxed{27.6°}$ (b) $P_{out}=100\times10^6\times0.8=80$ MW $\omega_m=2\pi\times50=100\pi$ rad/s (2-pole) $\tau_{in}=P/\omega=80\times10^6/100\pi=\boxed{254.6\ kN\cdot m}$ (c) $P_{max}=\frac{3V_\varphi E_A}{X_S}=\frac{3\times8314\times15658}{2.281}=\boxed{171.6\ MW}$ (at δ=90°) *Operating power (80 MW) is 46.6% of P_{max} → stable operating point with good margin.*
L5 Sync **Q38.** A Δ-connected, 60 Hz, 4-pole synchronous generator: V_T=480 V, X_S=0.1 Ω, R_A=0.015 Ω. Full load: I_L=1200 A at 0.8 PF lagging. Friction/windage=40 kW, core losses=30 kW, field losses negligible. (a) Find shaft rotation speed (b) Find full-load E_A (c) Find overall efficiency	**A38.** (a) $n_s=120\times60/4=\boxed{1800\ rpm}$ Δ-connected: $V_\varphi=V_T=480$ V. $I_A=I_L/\sqrt{3}=1200/\sqrt{3}=692.8\ A\angle-36.87°$ (b) $E_A=480+(554.2-j415.7)(0.015+j0.1)$ $=480+(554.2\times0.015+554.2\times j0.1+(-j415.7)\times0.015+(-j415.7)(j0.1))$ $=480+(8.31+j55.42-j6.24+41.57)=480+49.88+j49.18=529.9+j49.18$ $\lvert E_A\rvert=\sqrt{529.9^2+49.18^2}=\boxed{532.2\ V}$ (c) $P_{out}=\sqrt{3}\times480\times1200\times0.8=798.6$ kW $P_{in}=P_{out}+P_{Cu}+P_{core}+P_{fw}$ $P_{Cu}=3\times692.8^2\times0.015=21.6$ kW $P_{in}=798.6+21.6+30+40=890.2$ kW $\eta=\frac{798.6}{890.2}=\boxed{89.7\%}$

SECTION F — SYNCHRONOUS MOTOR

L5 Sync **Q39.** A 208 V, 45 kVA, Δ-connected, 60 Hz synchronous motor: X_S=2.5 Ω, R_A≈0. Losses: P_{fw}=1.5 kW, P_{core}=1.0 kW. Shaft load=15 hp at PF=0.80 leading. Find I_A, I_{Line}, E_A.	**A39.** $P_{in}=15\times746+1500+1000=13690$ W $V_\varphi=208$ V (Δ) $I_A=\frac{13690}{3\times208\times0.80}=27.4$ A leading → $I_A=27.4\angle+36.87°$ $I_{Line}=\sqrt{3}\times27.4=\boxed{47.5\ A}$ $E_A=208-j2.5\times27.4\angle36.87°=208-68.5\angle126.87°=208+41.1-j54.8=249.1-j54.8$ $\lvert E_A\rvert=\sqrt{249.1^2+54.8^2}=\boxed{255.1\ V}$ (>V_φ → consistent with leading/overexcited)
L5 Sync **Q40.** Using Q39 motor ($\lvert E_A\rvert$=255.1 V fixed). Shaft load increases to **30 hp**. Find new δ, I_A, I_{Line}, and power factor.	**A40.** $P_{new}=30\times746+2500=24880$ W $\sin\delta_{new}=\frac{P_{new}\times X_S}{3V_\varphi\lvert E_A\rvert}=\frac{24880\times2.5}{3\times208\times255.1}=0.391$ → $\delta=\boxed{23.0°}$ $E_{A,new}=255.1\angle-23.0°=234.8-j99.7$ $I_{A,new}=\frac{208-(234.8-j99.7)}{j2.5}=\frac{-26.8+j99.7}{j2.5}=39.9+j10.7$ $\lvert I_{A,new}\rvert=\boxed{41.3\ A}$ $I_{Line}=\boxed{71.5\ A}$ PF=cos(arctan(10.7/39.9))=$\boxed{0.966\ leading}$

L5 Sync	**Q41.** A 208 V, 45 kVA, Δ-connected motor (X_S=2.5 Ω). Shaft=15 hp, PF=0.85 lagging initially. Field current then **increased by 25%** (E_A increases 25%). Find new I_A and PF after field increase.	**A41.** P_{in}=13690 W, $I_{A,old}$=13690/(3×208×0.85)=25.8 A∠−31.79° $E_{A,old}$=208−j2.5×25.8∠−31.79°=174.0−j54.8 → $\|E_{A,old}\|$=182.4 V $E_{A,new}$=1.25×182.4=228.0 V. P const → $E_A$sinδ=54.8 const $\sin\delta_{new}$=54.8/228.0=0.2404 → δ_{new}=13.9° → $E_{A,new}$=228.0∠−13.9°=221.3−j54.8 $I_{A,new}=\frac{208-(221.3-j54.8)}{j2.5}$=21.9+j5.3 → $\|I_{A,new}\|$= $\boxed{22.5\text{ A}}$ PF= $\boxed{0.972\text{ leading}}$ *PF shifted from 0.85 lagging → 0.972 leading by increasing field current 25%. Motor now supplies Q to grid.*
L5 Sync	**Q42.** A 380 V, 60 Hz, 4-pole, Y-connected synchronous motor draws 30 A at unity PF (lossless). (a) Find output torque (b) To change PF to 0.8 leading — increase or decrease I_F? Explain with phasor reasoning. (c) Find new I_L at 0.8 leading PF.	**A42.** (a) P=√3×380×30×1.0=19752 W ω_m=2π×1800/60=60π rad/s τ=19752/(60π)= $\boxed{104.8\text{ N·m}}$ (b) **Increase I_F**. At unity PF, I_A is in phase with V_φ and $E_A$sinδ is fixed by load power. Increasing I_F → increases $\|E_A\|$ while $E_A$sinδ = constant → δ decreases. From motor KVL $E_A=V_\varphi-jX_SI_A$: larger $\|E_A\|$ with smaller δ forces I_A to rotate toward leading. The machine becomes overexcited. (c) $I_{L,new}=\frac{P}{\sqrt{3}\times V_T\times PF}=\frac{19752}{\sqrt{3}\times380\times0.8}=$ $\boxed{37.5\text{ A}}$
L5 Sync	**Q43.** A 3.3 kV, Y-connected synchronous motor: X_S=5 Ω, R_A≈0. At rated load, δ=30°, $\|E_A\|$=2400 V. (a) Find the real power P developed per phase (b) Find the pull-out torque (n_s=1000 rpm, 6-pole, 50 Hz) (c) How much safety margin does the current operating point have before pull-out?	**A43.** V_φ=3300/√3=1905 V ω_m=2π×1000/60=104.7 rad/s (a) $P_{phase}=\frac{V_\varphi E_A\sin\delta}{X_S}=\frac{1905\times2400\times\sin30°}{5}=\frac{1905\times2400\times0.5}{5}=$ $\boxed{457.2\text{ kW/phase}}$ Total P=3×457.2=1371.6 kW (b) $\tau_{max}=\frac{3V_\varphi E_A}{X_S\omega_m}=\frac{3\times1905\times2400}{5\times104.7}=$ $\boxed{26.19\text{ kN·m}}$ (c) Current torque $\tau=P_{total}/\omega_m$=1371600/104.7=13.1 kN·m Safety margin=τ_{max}/τ=26.19/13.1= $\boxed{2.0\times\text{ (100\% margin — very safe)}}$

SECTION G — INDUCTION MACHINE CALCULATIONS

L6 IM	**Q44.** A 208 V, 10 hp, 4-pole, 60 Hz, Y-connected induction motor, full-load slip=5%. Find: (a) n_s, (b) n_m, (c) f_r, (d) shaft torque τ_{shaft}.	**A44.** (a) n_s=120×60/4= $\boxed{1800\text{ rpm}}$ (b) n_m=(1−0.05)×1800= $\boxed{1710\text{ rpm}}$ (c) f_r=0.05×60= $\boxed{3\text{ Hz}}$ (d) ω_m=2π×1710/60=179.1 rad/s τ=7460/179.1= $\boxed{41.7\text{ N·m}}$
L6 IM	**Q45.** A 380 V, 4-pole, 50 Hz, Y-connected, 80 hp induction motor: R_1=0.10 Ω, R_2=0.07 Ω, X_M=10 Ω, $X_1=X_2$=0.21 Ω, P_{mech}=1.5 kW, P_{core}=1.0 kW. Slip s=0.10. Find I_L, PF, P_{AG}, τ_{ind}, P_{conv}, P_{out}, τ_{load}, η, and rotor speed.	**A45.** n_s=1500 rpm, ω_s=50π rad/s, n_m=1350 rpm, ω_m=45π rad/s, V_φ=220 V Z_F=j10‖(0.7+j0.21)=0.668+j0.252 Z_{total}=0.768+j0.462 $\|Z_{total}\|$=0.896 Ω I_L=220/0.896= $\boxed{245\text{ A}}$ PF=cos(arctan(0.462/0.768))= $\boxed{0.857\text{ lag}}$ P_{in}=3×220×245×0.857=139 kW P_{SCL}=3×245²×0.10=18.0 kW P_{AG}=139−18−1= $\boxed{120\text{ kW}}$ τ_{ind}=120000/(50π)= $\boxed{764\text{ N·m}}$ P_{conv}=0.9×120= $\boxed{108\text{ kW}}$ P_{out}=108−1.5= $\boxed{106.5\text{ kW}}$ τ_{load}=106500/(45π)= $\boxed{753\text{ N·m}}$ η=106.5/139= $\boxed{76.6\%}$

L6 IM

Q46. A 480 V, 60 Hz, 50 hp induction motor draws 60 A at 0.85 PF lagging. Known losses: P_{SCL}=2 kW, P_{RCL}=700 W, P_{fw}=600 W, P_{core}=1800 W. Find: (a) P_{AG}, (b) slip s, (c) P_{conv}, (d) P_{out}, (e) η.

A46.

$P_{in}=\sqrt{3}\times480\times60\times0.85=42{,}416$ W

(a) $P_{AG}=42416-2000-1800=\boxed{38{,}616\text{ W}}$

(b) $s=P_{RCL}/P_{AG}=700/38616=\boxed{1.81\%}$

(c) $P_{conv}=(1-0.0181)\times38616=\boxed{37{,}916\text{ W}}$

(d) $P_{out}=37916-600=\boxed{37{,}316\text{ W}\approx50.0\text{ hp}\ ✓}$

(e) $\eta=37316/42416=\boxed{88.0\%}$

L6 IM

Q47. A 460 V, 25 hp, 60 Hz, 4-pole, Y-connected wound-rotor IM: R_1=0.641 Ω, R_2=0.332 Ω, X_1=1.106 Ω, X_2=0.464 Ω, X_M=26.3 Ω, P_{rot}=1100 W. Find: (a) s_{max} and speed at max torque, (b) τ_{max}, (c) starting torque at s=1.

A47.

V_φ=265.6 V, $\omega_s=60\pi=188.5$ rad/s, $V_{TH}=265.6\times26.3/27.406=254.9$ V, $R_{TH}=0.591$ Ω, $X_{TH}=1.106$ Ω

(a) $s_{max}=\dfrac{0.332}{\sqrt{0.591^2+1.570^2}}=\dfrac{0.332}{1.677}=\boxed{0.198}$ $\quad n_{max}=(1-0.198)\times1800=\boxed{1444\text{ rpm}}$

(b) $\tau_{max}=\dfrac{3\times254.9^2}{2\times188.5\times(0.591+1.677)}=\dfrac{194960}{855.5}=\boxed{228\text{ N·m}}$

(c) $\tau_{start}=\dfrac{3\times254.9^2\times0.332}{188.5\times[(0.591+0.332)^2+1.570^2]}=\dfrac{64732}{625}=\boxed{104\text{ N·m}}$

L6 IM

Q48. For the motor in Q47: rotor resistance doubled (R_2=0.664 Ω, external resistance inserted via slip rings). (a) New s_{max} and speed? (b) New τ_{max}? (c) New starting torque? (d) State the general rules for R_2's effect on induction motor performance.

A48.

(a) $s_{max}\propto R_2\rightarrow s_{max,new}=2\times0.198=\boxed{0.396}$ $\quad n=(1-0.396)\times1800=\boxed{1087\text{ rpm}}$

(b) τ_{max} independent of $R_2\rightarrow\boxed{228\text{ N·m (unchanged)}}$

(c) $\tau_{start,new}=\dfrac{3\times254.9^2\times0.664}{188.5\times[(0.591+0.664)^2+1.570^2]}=\dfrac{129461}{761}=\boxed{170\text{ N·m}\uparrow}$

(d) Rules:

- $s_{max}=\dfrac{R_2}{\sqrt{R_{TH}^2+(X_{TH}+X_2)^2}}$ — directly proportional to R_2
- τ_{max} — completely independent of R_2
- τ_{start} increases as R_2 increases, reaching maximum when s_{max}=1 (i.e. $R_2=\sqrt{R_{TH}^2+(X_{TH}+X_2)^2}$)
- Higher R_2 → lower efficiency (more rotor copper loss at same slip)

L6 IM

Q49. A 7.5 hp, 4-pole, 208 V, 60 Hz, Y-connected induction motor (rated 28 A) is tested:
DC: V_{DC}=13.6 V, I_{DC}=28 A
No-load: V_T=208 V, I_A=8.17 A, P_{NL}=420 W
LR test: V_T=25 V, I_A=28 A, P_{LR}=920 W, f_{test}=15 Hz
Find R_1, R_2, X_1, X_2, X_M and compute τ_{max}.

A49.

$R_1=13.6/(2\times28)=\boxed{0.243\ \Omega}$

$X_1+X_M\approx120.1/8.17=14.7\ \Omega\rightarrow\boxed{X_M\approx14.7\ \Omega}$

$R_{LR}=920/(3\times28^2)=0.391\ \Omega\quad Z_{LR}=(25/\sqrt{3})/28=0.516\ \Omega\quad X_{LR,15Hz}=\sqrt{0.516^2-0.391^2}=0.334\ \Omega$

$X_{LR,60Hz}=0.334\times(60/15)=1.336\ \Omega\rightarrow X_1=X_2=\boxed{0.668\ \Omega}$ (Design A, equal split)

$R_2=0.391-0.243=\boxed{0.148\ \Omega}$

V_{TH}=114.8 V, R_{TH}=0.203 Ω, $s_{max}=0.148/1.367=0.108$

$\tau_{max}=3\times114.8^2/[2\times(60\pi)(0.203+1.367)]=\boxed{66.8\text{ N·m}}$

L6 IM

Q50. A 415 V, 50 Hz, 4-pole, 3-phase induction motor runs at full load. Supply voltage suddenly drops to **0.9×415 V** (10% voltage sag).
(a) What happens to the maximum torque capability?
(b) If the load demands rated torque, what happens to the motor?
(c) Quantify the change in τ_{max} and explain using the formula.

A50.
(a) From τ_{max} formula: $\tau_{max} \propto V_{TH}^2 \propto V_1^2$:
$\tau_{max,new}=\tau_{max,old}\times(0.9)^2=0.81\times\tau_{max,old}$
→ $\boxed{\textbf{19\% reduction in } \tau_{max}}$
(b) If load still requires rated torque T_{rated}, and τ_{max} dropped to $0.81\times\tau_{max,old}$: as long as $\tau_{rated} < \tau_{max,new}$, the motor continues running but at **higher slip** (operating point moves down-left on torque-speed curve toward the peak). If $\tau_{rated} > \tau_{max,new}$, the motor cannot produce enough torque → it **stalls (pulls out)**.
(c) The formula shows $\tau_{max} = 3V_{TH}^2/[2\omega_s(R_{TH}+...)]$ — numerator has $V_{TH}^2 \propto V_{supply}^2$. Doubling voltage would quadruple τ_{max}; reducing by 10% reduces by 19%.
This is why voltage sags during motor starting of large motors can stall neighbouring motors on the same bus. Also why voltage support (capacitor banks, synchronous condensers) is critical in industrial plants.

L6 IM

Q51. Motor classes for design induction motors: briefly describe Classes **A, B, C, D** in terms of starting torque, starting current, and slip. Give one application for each.

A51.

Class	Starting Torque	Starting Current	Full-load Slip	Application
A	Normal (100–150%)	High (6–7×I_{rated})	Low (<5%)	Fans, centrifugal pumps, compressors — low inertia, quick start
B	Normal (≥150%)	Lower (≤5×I_{rated})	Normal (≤5%)	Machine tools, large fans, conveyor drives — where starting current matters
C	High (≥200%)	Low (≤5×I_{rated})	Normal	Crushers, reciprocating compressors, refrigeration — heavy starting loads
D	Very high (≥275%)	Low	High (5–20%)	Punch presses, bulldozers, shearing machines — shock loads; high-slip for energy storage in flywheel

Class A/B: R_2 low → s_{max} near 0 → efficient, runs near sync speed. Class D: R_2 high → s_{max} near 1 → high starting torque, runs at high slip (low speed, less efficient).

L6 IM

Q52. A wound-rotor induction motor has the Thevenin circuit equivalent: V_{TH}=200 V, R_{TH}=0.3 Ω, X_{TH}=1.0 Ω, X_2=0.5 Ω, R_2=0.2 Ω, ω_s=188.5 rad/s.
(a) Find R_{ext} needed for **maximum starting torque** (s_{max}=1)
(b) Verify by computing starting torque with and without R_{ext}

A52.
(a) For $s_{max}=1$: $R_2+R_{ext} = \sqrt{R_{TH}^2+(X_{TH}+X_2)^2}$:
$\sqrt{0.3^2+(1.0+0.5)^2}=\sqrt{0.09+2.25}=\sqrt{2.34}=1.530\ \Omega$
$R_{ext}=1.530-R_2=1.530-0.200=\boxed{\textbf{1.330 Ω per phase}}$
(b) Without R_{ext} (R_2=0.2, s=1):
$$\tau_{start}=\frac{3\times200^2\times0.2}{188.5\times[(0.3+0.2)^2+1.5^2]}=\frac{24000}{188.5\times2.5}=\boxed{\textbf{50.8 N·m}}$$
With R_{ext} (total R=1.530 Ω, s=1):
$$\tau_{start,ext}=\frac{3\times200^2\times1.530}{188.5\times[(0.3+1.530)^2+1.5^2]}=\frac{183600}{188.5\times(3.348+2.25)}=\frac{183600}{1055.0}=\boxed{\textbf{174 N·m ↑ 3.4×}}$$

SECTION H — SPECIAL MACHINES

L7 Special **53.** A universal motor (series DC motor on AC): V_s=200 V_{rms}, 50 Hz, R_a=0.4 Ω, L_a=3 mH, R_f=0.8 Ω, L_f=6 mH. Full load: I_a=10 A, n=1000 rpm. At half torque: find new I_a and speed.

A53.

$X_{total}=2\pi\times50\times9\times10^{-3}=2.827\ \Omega$ $R_{total}=1.2\ \Omega$ $\omega_m=104.7$ rad/s

$200^2=(E_a+10\times1.2)^2+(10\times2.827)^2 \rightarrow E_a=\boxed{186\text{ V}}$ $K_eK_f=186/(10\times104.7)=0.1776$

Half torque: $\tau\propto I_a^2 \rightarrow I_{a,new}=10/\sqrt{2}=\boxed{7.07\text{ A}}$

$200^2=(E_{a,new}+8.48)^2+399.6 \rightarrow E_{a,new}=190.5$ V

$\omega_{new}=190.5/(0.1776\times7.07)=\boxed{151.8\text{ rad/s} \rightarrow 1449\text{ rpm}}$

L7 Special **54.** Describe the **120° conduction pattern** of a BLDC motor with a 3-phase inverter. Which upper and lower switches conduct in each of the 6 commutation intervals? Why is this called "brushless DC"?

A54.

The 3-phase inverter has 3 upper switches (S1, S3, S5) and 3 lower switches (S2, S4, S6). Hall sensors detect rotor position in 60° steps → 6 commutation intervals per electrical cycle. In each interval, exactly **one upper + one lower switch** conducts (always one from different phases):

Interval	Upper	Lower	Phases conducting
I (0–60°)	S1 (A)	S6 (B)	A→B (C open)
II (60–120°)	S1 (A)	S2 (C)	A→C (B open)
III (120–180°)	S3 (B)	S2 (C)	B→C (A open)
IV (180–240°)	S3 (B)	S4 (A)	B→A (C open)
V (240–300°)	S5 (C)	S4 (A)	C→A (B open)
VI (300–360°)	S5 (C)	S6 (B)	C→B (A open)

Each device conducts for 120° per cycle. **Called "brushless DC"** because: the DC source delivers a constant DC current to the two active phases at any moment (like a DC motor); from the battery's perspective, DC current is drawn and commutation is electronic (not mechanical). DC motor equations (E_a=kω, τ=kI) apply at any instant.

L7 Special **55.** Explain why a **single-phase induction motor** produces no starting torque. Then explain how **capacitor-start** and **shaded-pole** methods solve this problem.

A55.

No starting torque — double revolving field theory: A single-phase alternating current creates an alternating (pulsating) magnetic field, not a rotating one. This pulsating field can be mathematically decomposed into two equal counter-rotating fields of half magnitude. One rotates at $+n_s$, the other at $-n_s$. At standstill, both fields create equal and opposite torques that cancel exactly → net starting torque = 0. Once rotating, the field in the direction of rotation becomes dominant and the motor accelerates.

Capacitor-start method: An auxiliary winding is placed 90° (in space) from the main winding. A series capacitor shifts the auxiliary winding current ≈90° in time from the main winding current. Two currents 90° apart in space AND time → approximate rotating magnetic field → starting torque. A centrifugal switch disconnects the auxiliary winding at ~75% rated speed.

Shaded-pole method: A copper short-circuit ring is placed over part of each salient stator pole. The ring acts as a shorted secondary — it carries an induced current that opposes the changing flux in its portion of the pole, causing that part's flux to lag behind the unshaded portion's flux. This creates a "sweeping" field from unshaded to shaded region → weak rotating field → small starting torque. Simplest and cheapest; efficiency is low; used for very small motors (fans, clocks).

L7 Special **56.** Describe the **SRM (Switched Reluctance Machine)** energy conversion cycle using the inductance L(θ) curve. Why must the phase current be switched off before full alignment? What is the consequence of late switching?

A56.

Energy conversion using L(θ): The inductance of one stator phase varies with rotor position — minimum when poles are unaligned, maximum when fully aligned. The torque is:

$$\tau = \frac{1}{2} \cdot I^2 \cdot \frac{dL}{d\theta}$$

When the phase is energised at an unaligned position and $dL/d\theta > 0$ (inductance increasing), torque is positive (motoring). When $dL/d\theta < 0$ (inductance decreasing — rotor past aligned position), torque would be negative (braking/generating).

Must switch off before full alignment: If current remains on past the aligned position, $dL/d\theta$ becomes negative → torque reverses direction → the motor fights itself, wasting energy and causing torque ripple and backward impulses.

Consequence of late switching: The rotor continues past alignment with current still flowing → negative (braking) torque pulse. This increases torque ripple and reduces average torque and efficiency. In worst case, motor can fail to rotate or reverse. The commutation advance angle must be optimised for each speed (firing angle control).

SECTION I — ANALYSIS, DERIVATION & TRUE/FALSE

Mixed **Q57.** State whether each of the following is TRUE or FALSE and justify briefly:

(a) A DC series motor can be safely operated with no mechanical load.
(b) Increasing R_2 in an induction motor increases its maximum torque.
(c) An overexcited synchronous motor supplies reactive power to the grid.
(d) The air-gap power $P_{AG} = \tau_{ind} \cdot \omega_s$.
(e) A synchronous motor can start direct-on-line.
(f) Swapping any two supply phases of a 3-phase induction motor reverses its direction.
(g) The back-EMF in a BLDC motor is sinusoidal.
(h) Fractional-pitch windings reduce both fundamental voltage and harmonic voltages.

A57.

(a) **FALSE** — $\Phi \propto I_a$; no load → $I_a \to 0$ → $\Phi \to 0$ → speed → ∞. Mechanically catastrophic.

(b) **FALSE** — τ_{max} is independent of R_2. Only s_{max} shifts proportionally to R_2.

(c) **TRUE** — Overexcited: I_A leads V_φ → motor acts as capacitive load → supplies Q (reactive power) to grid. Used as synchronous condenser for power factor correction.

(d) **TRUE** — By definition: P_{AG} is the power transferred across air gap = $\tau_{ind} \cdot \omega_s$. Note $P_{conv} = \tau_{ind} \cdot \omega_m \neq P_{AG}$.

(e) **FALSE** — Cannot start DOL. Average starting torque = 0 because the stator field reverses torque direction twice per cycle at standstill.

(f) **TRUE** — Swapping 2 phases reverses the rotation direction of the stator's rotating magnetic field, reversing the motor.

(g) **FALSE** — BLDC has trapezoidal back-EMF (concentrated windings). Sinusoidal back-EMF is characteristic of distributed-winding synchronous AC motors.

(h) **PARTIALLY TRUE** — Fractional pitch reduces fundamental by factor $k_p < 1$, but reduces harmonic voltages much more (harmonic pitch factors are much smaller). Net effect is improved waveform quality.

L5 Sync **Q58.** A synchronous machine: $X_S = 1\ \Omega$, $R_A \approx 0$, $V_\varphi = 240\ V\angle 0°$, $|E_A| = 280$ V, $\delta = 30°$.

(a) Motor or generator? Why?
(b) Find P and Q per phase
(c) Over- or underexcited? Justify.

A58.

(a) $\delta = 30°$ with E_A defined as $E_A\angle\delta$: if E_A leads V_φ → **generator** (E_A ahead → pushes current out). If E_A lags V_φ → motor. At $\delta = +30°$ (E_A leads) → **Generator**.

(b) $$P_{phase} = \frac{V_\varphi E_A \sin\delta}{X_S} = \frac{240 \times 280 \times 0.5}{1} = \boxed{33{,}600 \text{ W/phase}} \quad P_{3\phi} = 100.8 \text{ kW}$$

$$I_A = \frac{E_A\angle 30° - V_\varphi\angle 0°}{jX_S} = \frac{280(\cos 30° + j\sin 30°) - 240}{j} = \frac{242.5 + j140 - 240}{j} = \frac{2.5 + j140}{j} = 140 - j2.5 \text{ A}$$

$$Q_{phase} = V_\varphi I_A \sin\theta = 240 \times 2.5 = \boxed{600 \text{ VAR/phase (positive, supplies Q)}}$$

(c) $|E_A| \cos\delta = 280 \times \cos 30° = 242.5$ V > $V_\varphi = 240$ V → **Overexcited generator** (supplies reactive power to grid).

L3 DC

Q59. Derive the torque-speed equation for a series DC motor from first principles, showing that torque ∝ (1/speed)² for constant supply voltage.

A59.

For series motor: $I_f = I_a = I$, and $\Phi = K_f I$. Starting from:

$$V_T = E_a + I(R_a + R_f) = K_e K_f I \omega_m + I \cdot R_s$$

$$\tau = K_e \Phi I = K_e K_f I^2 \rightarrow I = \sqrt{\frac{\tau}{K_e K_f}}$$

Substitute into KVL:

$$V_T = K_e K_f \cdot \sqrt{\frac{\tau}{K_e K_f}} \cdot \omega_m + R_s \cdot \sqrt{\frac{\tau}{K_e K_f}}$$

$$\omega_m = \frac{V_T}{\sqrt{K_e K_f \cdot \tau}} - \frac{R_s}{K_e K_f}$$

So for large τ (ignoring R_s term): $\omega_m \propto 1/\sqrt{\tau} \rightarrow \tau \propto 1/\omega_m^2$. This is the hyperbolic torque-speed characteristic of series motor — huge torque at low speed, very high speed at low torque.

This means $\tau \propto I_a^2$ (best starting torque per amp) but $\omega \rightarrow \infty$ as $\tau \rightarrow 0$. Applications: traction drives, cranes — loads that always have torque when running.

L6 IM

Q60. What happens if a running induction motor is **plugged** (two supply phases are swapped while motor runs at rated speed)?

(a) Explain what happens to slip and the direction of torque immediately after plugging.

(b) Why is plugging used in practice despite being stressful on the motor?

(c) Compare braking current during plugging vs normal starting.

A60.

(a) Before plugging: motor runs at $n_m \approx (1-0.03)n_s = 0.97n_s$ in the forward direction. After phase swap, the stator field now rotates at $-n_s$ (reverse direction). Relative speed of rotor to stator field = $n_m - (-n_s) = n_m + n_s \approx 1.97n_s$. New effective slip:

$$s_{plug} = \frac{n_s - n_m}{n_s} = \frac{-n_s - (+n_m)}{-n_s} = \frac{n_s + n_m}{n_s} \approx 2 - s_{rated} \approx \boxed{1.97}$$

The induced torque is now **in the reverse direction** — opposing the forward motion. The motor decelerates rapidly. A relay or controller must disconnect the motor once it reaches zero speed (otherwise it will restart in reverse).

(b) **Plugging** gives the fastest stopping time — deceleration torque is 1.5–2× the accelerating torque. Used in machine tools, hoists, and positioning systems where rapid stop is required. Much faster than coasting or DC injection braking for high-inertia loads.

(c) Plugging current ≈ 2× starting current (since effective voltage across circuit is ≈ $2V_T$ at $s \approx 2$). Very high thermal stress — plugging is only used intermittently and motor must be rated for it. Starting current ≈ $6 \times I_{rated}$, plugging current ≈ 10–$12 \times I_{rated}$.

L6 IM

Q61. A 3-phase, 4-pole, 50 Hz induction motor measured at steady state:

P_{in}=40 kW, P_{SCL}=1.5 kW, P_{core}=0.5 kW, P_{RCL}=1.9 kW, P_{fw}=0.6 kW.

Find: (a) slip, (b) n_m, (c) P_{out}, (d) η, (e) τ_{ind}, (f) τ_{load}.

A61.

(a) $P_{AG} = 40 - 1.5 - 0.5 = 38$ kW $s = P_{RCL}/P_{AG} = 1.9/38 = \boxed{5.0\%}$

(b) $n_s = 1500$ rpm $n_m = (1-0.05) \times 1500 = \boxed{1425 \text{ rpm}}$

(c) $P_{conv} = (1-0.05) \times 38 = 36.1$ kW $P_{out} = 36.1 - 0.6 = \boxed{35.5 \text{ kW (47.6 hp)}}$

(d) $\eta = 35.5/40 = \boxed{88.75\%}$

(e) $\omega_s = 50\pi$ rad/s $\tau_{ind} = 38000/(50\pi) = \boxed{241.7 \text{ N·m}}$

(f) $\omega_m = 2\pi \times 1425/60 = 149.2$ rad/s $\tau_{load} = 35500/149.2 = \boxed{238.0 \text{ N·m}}$

L5 Sync

Q62. Explain the concept of the **brushless exciter** used in large synchronous generators. Why is it preferred over slip rings and brushes for large machines? Draw a block diagram description.

A62.

Brushless exciter system block description:

1. **Pilot exciter** (small AC generator with PM rotor, on same shaft) → produces AC output regardless of external power.
2. **Main exciter** (AC generator: field on stator, armature on rotor shaft) → stator field current controlled by pilot exciter or AVR. Rotor armature produces AC at shaft rotation frequency.
3. **Rotating rectifier** (diode bridge, also on shaft) → rectifies main exciter AC output to DC.
4. **DC field current** → delivered directly to main synchronous generator field winding (also on shaft) — no slip rings or brushes needed anywhere.

Why preferred for large machines:

- Eliminates brush wear — no maintenance interruption for large machines in remote locations
- No brush voltage drop (significant loss for large field currents, e.g. 1000 A × 2V brush drop = 2 kW)
- No sparking in hazardous environments
- More reliable — brushes can fail and short-circuit or open-circuit the field
- Scalability — field current can be hundreds of amps without brush contact issues

Slip rings are still used for small/medium synchronous machines where simplicity and cost matter more than maintenance reduction.

L5 Sync

Q63. A 6-pole, 50 Hz, 3.3 kV, Y-connected synchronous motor drives a compressor at rated load. X_S=8 Ω, R_A≈0. P_{in}=2 MW at PF=0.9 lagging. (a) Find I_A, E_A, δ. (b) The load is suddenly removed (P→0) but field maintained. What is the new armature current and PF?

A63.

$V_\varphi=3300/\sqrt{3}=1905$ V

(a) $I_A=\dfrac{2\times10^6}{3\times1905\times0.9}=389.9\ \text{A}\angle-25.84°$ (lagging → motor underexcited)

$E_A=1905-j8\times389.9\angle-25.84°=1905-3119\angle64.16°=1905-(1357+j2806)=548-j2806$

$|E_A|=\sqrt{548^2+2806^2}=\boxed{2859\text{ V}}$ $\delta=\tan^{-1}(2806/548)=\boxed{79.0°\text{ (lagging } E_A\text{: motor mode ✓)}}$

(b) Load removed → P=0 → $E_A\sin\delta=0$ → δ=0 (E_A aligns with V_φ). $|E_A|$=2859 V maintained:

$I_{A,NL}=\dfrac{V_\varphi-E_A\angle0°}{jX_S}=\dfrac{1905-2859}{j8}=\dfrac{-954}{j8}=j119.3$ A

$|I_{A,NL}|=\boxed{119.3\text{ A}}$ Purely imaginary (j) → $\boxed{\text{PF=0 leading}}$ (purely reactive, supplying Q to grid)

With no real load, overexcited motor draws 119 A of purely leading reactive current — acting as a synchronous condenser. The 26% of rated current maintains a large reactive power supply to the grid.

L6 IM

Q64. Efficiency limit from slip: Show that for an induction motor, the maximum possible efficiency is bounded by $\eta_{max} \le (1-s)\times(P_{conv}/P_{AG})$. If slip=4%, friction/windage losses=2% of P_{conv}, and stator losses=8% of P_{in}, estimate overall efficiency.

A64.

Derivation:

$P_{conv} = (1-s)\cdot P_{AG}$

$P_{out} = P_{conv}-P_{rot} \le P_{conv} = (1-s)P_{AG} \le (1-s)P_{in}$

$\eta = \dfrac{P_{out}}{P_{in}} \le (1-s)$ (upper bound when all other losses → 0)

Numerical estimate: s=0.04 → $P_{conv}=0.96P_{AG}$. Let P_{in}=100 units.

P_{SCL}=8 units (stator) P_{core}≈2 units (estimate)

P_{AG}=100−8−2=90 units

P_{RCL}=0.04×90=3.6 units P_{conv}=0.96×90=86.4 units

P_{rot}=0.02×86.4=1.73 units P_{out}=86.4−1.73=84.67 units

$\eta=84.67/100=\boxed{84.7\%}$ (bound from slip alone: 1−s=96%)

Actual η < (1−s) because stator losses, core losses, and rotational losses all further reduce output. The slip-based bound is tight only for small machines with dominant rotor losses.

L5+L6	**Q65. Design question:** A factory has a 1 MW, 0.75 PF lagging induction motor load on a 11 kV bus. The utility charges a penalty for PF below 0.90 lagging. The factory wants to install a **synchronous motor** (rated 200 kW, $X_S=2\ \Omega$ per phase, Y-connected, 11 kV) running overexcited to supply the needed reactive power. Find the required $\lvert E_A\rvert$ of the synchronous motor.

A65.

Step 1 — Find current reactive power absorbed by induction motors:

$P_{load}=1$ MW $PF_{old}=0.75$ lag $\theta_{old}=\cos^{-1}(0.75)=41.41°$

$Q_{load}=P\times\tan\theta_{old}=1\times10^6\times\tan41.41°=0.882$ MVAR (absorbed, lagging)

Step 2 — Required reactive power after PF correction to 0.90:

$\theta_{new}=\cos^{-1}(0.90)=25.84°$ $Q_{required}=1\times10^6\times\tan25.84°=0.484$ MVAR

$Q_{sync\ motor\ must\ supply}=Q_{load}-Q_{required}=0.882-0.484=$ **0.398 MVAR leading**

Step 3 — Find I_A of sync motor (200 kW real power + 398 kVAR leading):

$$V_\varphi=11000/\sqrt{3}=6351\text{ V}\quad I_A=\frac{P+jQ_{supply}}{3V_\varphi}=\frac{200000+j398000}{3\times6351}=10.5+j20.9\text{ A}$$

(Leading current has positive imaginary part: $I_A=10.5+j20.9$ A, $\lvert I_A\rvert=23.4$ A)

Step 4 — Find E_A:

$E_A=V_\varphi-jX_S\cdot I_A=6351-j2\times(10.5+j20.9)=6351-j21+41.8=6392.8-j21$

$\lvert E_A\rvert=\sqrt{6392.8^2+21^2}=$ **6392.8 V ≈ 6.39 kV per phase** → Line: $6.39\times\sqrt{3}=11.07$ kV

FORMULA SHEET — EE3124 Electric Machines & Drives (All Topics)

1. FUNDAMENTAL CONSTANTS & CONVERSIONS

$\mu_0 = 4\pi \times 10^{-7}$ H/m (permeability of free space)

1 hp = 746 W

ω [rad/s] = $2\pi n$[rpm]/60

n [rpm] = $60\omega/(2\pi)$

$P = \tau\cdot\omega$ [W]; $\tau = P/\omega$ [N·m]

$\sqrt{2} \approx 1.4142$; $\sqrt{3} \approx 1.7321$; $\pi \approx 3.14159$

2. MAGNETIC CIRCUIT

Ampere's Law: $\oint H\cdot dl = \Sigma i \rightarrow H\cdot l_c = N\cdot i$

B–H relation: $B = \mu_0\mu_r\cdot H$ [T]

Flux: $\Phi = B\cdot A$ [Wb]

mmf: $\mathscr{F} = N\cdot i$ [A·turns]

Reluctance: $\mathfrak{R} = l/(\mu_0\mu_r A)$ [A·t/Wb]

Air gap: $\mathfrak{R}_g = g/(\mu_0 A_g)$

Ohm's law: $\Phi = \mathscr{F}/\mathfrak{R} = N\cdot i/\mathfrak{R}$

Series: $\mathfrak{R}_{total} = \mathfrak{R}_1+\mathfrak{R}_2+\ldots$ Parallel: $1/\mathfrak{R}_{total}=\Sigma(1/\mathfrak{R}_k)$

Flux linkage: $\lambda = N\cdot\Phi$ [Wb·turn]

Inductance: $L = \lambda/i = N^2/\mathfrak{R}$ [H]

Faraday: $e = -N\cdot d\Phi/dt = L\cdot di/dt$ [V]

Moving bar: $e = B\cdot L\cdot v$ [V]; $F = B\cdot I\cdot l$ [N]

Energy: $W = \frac{1}{2}Li^2 = \frac{1}{2}\Phi^2\mathfrak{R} = \frac{1}{2}BH(Vol)$ [J]

Hysteresis loss: $P_H = K_H\cdot f\cdot B_{max}{}^n$ ($n\approx1.6$–2)

Eddy current loss: $P_E = K_E\cdot f^2\cdot B_{max}{}^2$

Toroid: $B = \mu_0\mu_r\cdot NI/(2\pi r)$

KCL analogue: $\Sigma\Phi = 0$ at any node

KVL analogue: $\Sigma(Ni) = \Sigma(\Phi\cdot\mathfrak{R})$ around loop

3. THREE-PHASE POWER & CONNECTIONS

Y-connection: $V_L=\sqrt{3}\cdot V_\varphi$; $I_L=I_\varphi$

Δ-connection: $V_L=V_\varphi$; $I_L=\sqrt{3}\cdot I_\varphi$

Real power: $P = \sqrt{3}\cdot V_L\cdot I_L\cdot\cos\theta = 3\cdot V_\varphi\cdot I_\varphi\cdot\cos\theta$ [W]

Apparent power: $S = \sqrt{3}\cdot V_L\cdot I_L$ [VA]

Reactive power: $Q = S\cdot\sin\theta = P\cdot\tan\theta$ [VAR]

Power factor: PF = $\cos\theta = P/S$

Complex power: $\tilde{S} = P + jQ = \tilde{V}\cdot\tilde{I}^*$

Lagging PF: I lags V (inductive, Q>0 absorbed)

Leading PF: I leads V (capacitive, Q<0 or Q supplied)

4. SYNCHRONOUS SPEED

Sync speed: $n_s = 120\cdot f/P$ [rpm] (P = number of poles)

Angular: $\omega_s = 4\pi f/P$ [rad/s]

Elec–mech angle: $\theta_e = (P/2)\cdot\theta_m$

Elec–mech freq: $f_e = (P/2)\cdot n/60$

50 Hz table: 2P→3000, 4P→1500, 6P→1000, 8P→750, 20P→300 rpm

60 Hz table: 2P→3600, 4P→1800, 6P→1200, 8P→900 rpm

5. DC MACHINES

Back-EMF: $E_a = K_e\cdot\Phi\cdot\omega_m$ [V]

Torque: $\tau = K_e\cdot\Phi\cdot I_a$ [N·m] (same K_e!)

Flux (linear): $\Phi = K_f\cdot I_f$

Motor KVL: $V_a = E_a + I_a\cdot R_a$

Generator KVL: $V_a = E_a - I_a\cdot R_a$

Developed power: $P_d = E_a\cdot I_a = \tau\cdot\omega_m$

Shunt: $I_L=I_a+I_f$; $I_f=V_T/R_f$; $V_a=V_T$

Series: $I_L=I_a=I_f$; $V_T=E_a+I_a(R_a+R_f)$

T–ω (shunt/sep): $\omega = V_a/(K_e\Phi) - [R_a/(K_e\Phi)^2]\cdot\tau$

Series τ–ω: $\omega = V_T/\sqrt{(K_eK_f\cdot\tau)} - R_s/(K_eK_f)$

Series E ratio: $E_2/E_1 = (I_{a2}\cdot\omega_2)/(I_{a1}\cdot\omega_1)$

Losses: $P_{Cu}=I_a{}^2R_a$; $P_{brush}=V_{BD}\cdot I_a$; $P_f=I_f{}^2R_f$

Efficiency: $\eta = P_{out}/P_{in}$; $P_{in}=V_T\cdot I_L$

P_{out} (motor): $= P_d-P_{rot}-P_{core}$

6. AC MACHINE FUNDAMENTALS

3-phase currents: $i_a=I_M\sin(\omega t)$; $i_b=I_M\sin(\omega t-120°)$; $i_c=I_M\sin(\omega t-240°)$

Net field magnitude: $|B_{net}| = 1.5\cdot B_M$ (rotates at ω_e)

Peak coil voltage: $\hat{e} = N_C\cdot\omega_m\cdot\Phi_M$ [V]

RMS phase voltage: $V_{\varphi,rms} = \hat{e}/\sqrt{2}$

Winding factor: $k_w = k_p\cdot k_d \le 1$

Pitch factor: k_p = sin(pitch_angle/2)

7. SYNCHRONOUS MACHINES

Internal voltage: $E_A = K\cdot\Phi\cdot\omega$ [V rms]

Generator KVL: $E_A = V_\varphi + I_A(R_A+jX_S)$

Motor KVL: $V_\varphi = E_A + I_A(R_A+jX_S)$

Power (gen&motor): $P = 3\cdot V_\varphi\cdot E_A\cdot\sin\delta / X_S$

Max power (δ=90°): $P_{max} = 3\cdot V_\varphi\cdot E_A / X_S$

Reactive power: $Q = 3\cdot V_\varphi\cdot I_A\cdot\sin\theta$

Max torque (motor): $\tau_{max} = 3\cdot V_\varphi\cdot E_A/(X_S\cdot\omega_m)$

Torque general: $\tau_{ind} = k\cdot B_R\cdot B_{net}\cdot\sin\delta$

DC test (Y): $R_A = V_{DC}/(2\cdot I_{DC})$

OC test (Y): $E_A = V_{T,OC}/\sqrt{3}$

SC test: $Z_S = E_A/I_{A,SC}$; $X_S=\sqrt{(Z_S{}^2-R_A{}^2)}$

Voltage regulation: $VR\% = (V_{NL}-V_{FL})/V_{FL} \times 100\%$

V_{NL} (Y-conn): $= |E_A|$ (field current unchanged)

Phasor (Gen, V_φ=ref∠0°):

Lagging load: $I_A\angle-\theta \rightarrow |E_A|>V_\varphi \rightarrow VR>0$

Leading load: $I_A\angle+\theta \rightarrow |E_A|<V_\varphi \rightarrow VR<0$

10. KEY INEQUALITIES AND CONDITIONS

Sync generator mode: E_A leads V_φ ($\delta>0$)
Sync motor mode: E_A lags V_φ ($\delta<0$)
Overexcited (gen or motor): $|E_A|\cos\delta > V_\varphi$ → supplies Q (leading I_A)
Underexcited (gen or motor): $|E_A|\cos\delta < V_\varphi$ → absorbs Q (lagging I_A)
DC motor vs generator: $V_T>E_a$→motor; $E_a>V_T$→generator
Induction motor stability: operates at $s<s_{max}$ (rising torque region)
Grid sync (all 4): equal V, equal seq, equal angle, $f_{inc}>f_{grid}$
Series motor: NEVER run at no-load (speed → ∞)
Sync motor: CANNOT start DOL (zero average starting torque)
Max IM starting torque: when $R_2=\sqrt{R_{TH}^2+(X_{TH}+X_2)^2}$

11. POWER FACTOR CORRECTION (SYNC CONDENSER)

Reactive power demand: $Q_{load}=P\cdot\tan(\cos^{-1}(PF_{old}))$
After correction: $Q_{required}=P\cdot\tan(\cos^{-1}(PF_{new}))$
Q to supply: $Q_{supply}=Q_{load}-Q_{required}$
Sync motor current: $S_{SM}=P_{SM}+jQ_{supply}$
I_A of sync motor: $I_A=S_{SM}/(3V_\varphi)$
E_A needed: $E_A=V_\varphi-I_A\cdot jX_S$ (motor sign)
Induction motor PF: Always lagging (absorbs Q); cannot supply Q

12. MACHINE PARAMETER TEST SUMMARY

Sync Gen (3 tests):
DC: $R_A=V_{DC}/(2I_{DC})$ [Y-conn, 2 series windings]
OC: $E_A=V_{T,OC}/\sqrt{3}$ [Y]; $Z_S=E_A/I_{SC}$; $X_S=\sqrt{Z_S^2-R_A^2}$
Induction Motor (3 tests):
DC: $R_1=V_{DC}/(2I_{DC})$ [Y-conn]
NL: $X_M\approx V_{\varphi,NL}/I_{NL}$; $P_{rot}+P_{core}=P_{NL}-3I_{NL}^2R_1$
LR: $R_{LR}=P_{LR}/(3I^2)$; $Z_{LR}=V_\varphi/I$; scale X by f_{rated}/f_{test}

8. INDUCTION MACHINES

Slip: $s = (n_s-n_m)/n_s$; $n_m=(1-s)\cdot n_s$; $\omega_m=(1-s)\omega_s$
Rotor frequency: $f_r = s\cdot f_e$
Rotor quantities: $E_R=s\cdot E_{R0}$; $X_R=s\cdot X_{R0}$
Rotor current (referred): $I_2 = E_1/(R_2/s + jX_2)$
Power flow chain:
$P_{AG} = P_{in}-P_{SCL}-P_{core} = 3I_2^2R_2/s = \tau_{ind}\cdot\omega_s$
$P_{RCL} = s\cdot P_{AG} = 3I_2^2R_2$
$P_{conv} = (1-s)\cdot P_{AG} = \tau_{ind}\cdot\omega_m$
$P_{out} = P_{conv}-P_{rot}$; $\tau_{load} = P_{out}/\omega_m$
Key ratios: $P_{RCL}/P_{AG}=s$; $P_{conv}/P_{AG}=1-s$
Thevenin: $V_{TH}\approx V_1\cdot X_M/(X_1+X_M)$
$R_{TH}\approx R_1\cdot(X_M/(X_1+X_M))^2$; $X_{TH}\approx X_1$
Induced torque: $\tau_{ind} = 3V_{TH}^2(R_2/s)/[\omega_s\{(R_{TH}+R_2/s)^2+(X_{TH}+X_2)^2\}]$
Max torque slip: $s_{max} = R_2/\sqrt{R_{TH}^2+(X_{TH}+X_2)^2}$
Max torque: $\tau_{max} = 3V_{TH}^2/[2\omega_s(R_{TH}+\sqrt{R_{TH}^2+(X_{TH}+X_2)^2})]$
τ_{max} is INDEPENDENT of R_2; $s_{max} \propto R_2$
Torque ∝ V_1^2 (proportional to square of supply voltage)
DC test (Y): $R_1=V_{DC}/(2\cdot I_{DC})$
LR test: $R_{LR}=P/(3I^2)$; $Z_{LR}=V_\varphi/I$; $X_{LR}=\sqrt{Z^2-R^2}\cdot(f_r/f_t)$
$R_2=R_{LR}-R_1$; Design A/D: $X_1=X_2=X_{LR}/2$
Plugging: $s_{plug} \approx 2-s_{rated} \approx 2$ (immediately after phase reversal)

9. SPECIAL MACHINES

Universal motor voltage: $|V_s|^2=(E_a+I_aR_{total})^2+(I_aX_{total})^2$
$X_{total}=2\pi f(L_a+L_f)$; $R_{total}=R_a+R_f$
$\tau\propto I_a^2$ (series) → half torque: $I_{a,new}=I_{old}/\sqrt{2}$
BLDC: $V_s=E_a+2I\cdot R_{ph}$; $P_{conv}=2\cdot E\cdot I$
$E_a=k\cdot\omega$; $\tau=k\cdot I$ (DC equations apply)
SRM torque: $\tau=\frac{1}{2}\cdot I^2\cdot dL/d\theta$
Switch off current BEFORE full alignment ($dL/d\theta$ changes sign)